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A. V. Il'in (Moscow, Lomonosov Moscow State University (MSU)). **Invertibility of dynamical systems using higher-order sliding modes.**

In this contribution we investigate the problem of invertibility of linear time-invariant continuous dynamical systems i. e., the problem is to reconstruct the unknown input of a system using the measured output.

The main aim of this work is to solve the problem on the basis of stabilization methods with the use of higher-order sliding modes, which permits one to construct continuous controls $u(t)$ without additional filtering.

Consider the linear time-invariant dynamical system

$$\dot{x} = Ax + b\xi, \qquad y = cx, \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the unknown state vector of the system, $y(t) \in \mathbf{R}$ is the measured output, and $\xi(t) \in \mathbf{R}$ is the unknown input of the system; A, b, and c are known constant matrices.

We also require that the estimate of a continuous signal $\xi(t)$ should be continuous.

We use controlled model of system (1) in the form

$$\dot{\widetilde{x}} = A\widetilde{x} + bu, \qquad \widetilde{y} = cx,$$

where the control $u(t)$ is aimed at the null stabilization of the system for the deviations $e(t) = \widetilde{x}(t) - x(t)$ with measurable output $\varepsilon(t) = \widetilde{y}(t) - y(t)$; this system has the form

$$\dot{e} = A + b(u - \widetilde{\xi}), \qquad \varepsilon = ce.$$

We assume that system (1) satisfies the following conditions.

Condition 1. System (1) is controllable and observable, i. e., is in general position.

Condition 2. The function $\xi(t) \in \Omega^1\{|\xi(t)| \leqslant \xi^0, |\dot{\xi}(t)| \leqslant \xi^1\}$.

Condition 3. The invariant zeros of system (1) are in $\mathbf{C}_-$.

Condition 4. The system (1) has the first relative order, i.e., the condition $cb \neq 0$ is satisfied.

The system (1) can be reduced by a nonsingular transformation to a form in which the null dynamics is separated, then the observer would be represented as

$$\begin{cases} \dot{x}_1 = x_2, \\ \quad\vdots \\ \dot{x}_{n-1} = -\beta_1 x_1 - \cdots - \beta_{n-1} x_{n-1} + y, \\ \dot{y} = -\overline{a}_1 x_1 - \cdots - \overline{a}_{n-1} x_{n-1} - \overline{a}_n y + \xi(t). \end{cases} \tag{2}$$

Then the observer has the form

$$\begin{cases} \dot{\widetilde{x}}_1 = \widetilde{x}_2, \\ \quad \vdots \\ \dot{\widetilde{x}}_{n-2} = \widetilde{x}_{n-1}, \\ \dot{\widetilde{x}}_{n-1} = -\beta_1 \widetilde{x}_1 - \dots - \beta_{n-1}\widetilde{x}_{n-1} + y. \end{cases} \tag{3}$$

Consider the controlled model for the last equation of system (2),

$$\dot{\widetilde{y}} = -\overline{a}_1 \widetilde{x}_1 - \dots - \overline{a}_{n-1}\widetilde{x}_{n-1} - \overline{a}_n y + u.$$

To stabilize the error $\varepsilon(t) = \widetilde{y}(t) - y(t)$, one can use various methods of stabilization of systems under uncertainty. In particular, it was suggested in [1–3] to stabilize system (3) by the discontinuous control, we use a second-order sliding mode

$$\begin{aligned} u &= u_1 + u_2, \\ u_1 &= \begin{cases} -u_1, & \text{if} \quad |u_1| > \mu, \\ -\alpha_1 \operatorname{sgn}(\varepsilon(t)), & \text{if} \quad |u_1| \leqslant \mu, \end{cases} \\ u_2 &= -\lambda |\varepsilon|^{\rho} \operatorname{sgn}(\varepsilon(t)), \end{aligned} \tag{4}$$

where $\alpha_1 > \xi^1$, $\mu > \xi^0$, $\lambda > 0$, $\rho \in (0;1)$.

By virtue of the continuity of $u(t)$, the control itself can be used to estimate the unknown signal $\varepsilon(t)$; the estimation error (in the case of an ideal sliding mode) converges exponentially to zero, and the convergence rate is determined by the convergence rate of the observer (2).

Theorem. *Let Conditions* 1–4 *be satisfied for system* (1). *Then the observer* (3) *and the control* $u(t)$ *in* (4) *provide the asymptotic estimate* $\widetilde{\xi}(t) = u(t)$ *for the unknown input signal* $\xi(t)$, *starting from some time.*

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