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G. A. Sviridyuk, **S. A. Zagrebina**, **A. S. Konkina** (Chelyabinsk, South Ural State University). **Modified transport flow at the crossroads.**

УДК 517.9

Abstract: Currently, one of the important problems of the megalopolis is traffic management, and in connection with the problem of the formation of predatory and congestion situations in settlements, respectively, these studies are relevant. This paper describes a computational experiment for a mathematical model of a traffic flow.

Keywords:
Multipoint initial-final condition, geometric graph, traffic flows, Oskolkov equation.

Consider a finite ordered set $\mathbf{\Gamma} = \{\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_i, \ldots\}$ geometric graphs, where $\mathbf{G}_i = \mathbf{G}_i(\mathcal{V}_i, \mathcal{E}_i)$. Each pair of geometric graphs $\mathbf{G}_i$ и $\mathbf{G}_{i+1}$ corresponds to the i the intersection before and after changing the traffic signal. $\mathcal{V}_i = \{V_{ij}\}$ this is the set of vertices of a geometric graph $\mathbf{G}_i$ and $\mathcal{E}_i = \{E_{ik}\}$ it's a lot of edges $\mathbf{G}_i$, and each edge E_{ik} each column $\mathbf{G}_i$ matches two numbers — ≪ length ≫ edge $l_{ik} \in \mathbb{R}_+$ and ≪ width ≫ $d_{ik} \in \mathbb{R}_+$. Thus, the mathematical model has the form

$$\lambda_i u_{ikt} - u_{iktxx} = \nu_i u_{ikxx} + f_{ik}, \tag{1}$$

$$\begin{gathered} u_{ik}(0,t) = u_{im}(l_{im},t) = u_{il}(0,t) = u_{in}(l_{in},t), \\ \forall E_{ik} \in E^{\alpha}(V_{ij}),\ \forall E_{im} \in E^{\omega}(V_{ij}), \end{gathered} \tag{2}$$

$$\sum_{E_{ik} \in E^{\alpha}(V_{ij})} b_{ik} u_{ikx}(0,t) - \sum_{E_{im} \in E^{\omega}(V_{ij})} b_{im} u_{imx}(l_{im},t) = 0. \tag{3}$$

There $u_{ik} = u_{ik}(x,t)$, $x \in [0, l_{ik}]$, $t \in \overline{\mathbb{R}}_+$ $(\equiv \{0\} \cup \mathbb{R}_+)$ characterizes the average speed of the traffic flow on E_{ik} ; $f_{ik} = f_{ik}(x,t)$, $(x,t) \in [0, l_{ik}] \times \overline{\mathbb{R}}_+$, responds to the (average) force that makes the wheels of vehicles spin. The odds λ_i are equal to one, divided by the retardation coefficient, which can take negative values, so we consider $\lambda_i \in \mathbb{R}$. The odd ν_i is responsible for the viscosity of the traffic flow, i. e. for his ability ≪ extinguish ≫ sharp drops in speed; within the meaning of $\nu_i \in \mathbb{R}_+$. We will consider the traffic flow at an intersection with traffic lights, where the intersection is represented as an eight-edge graph.

The considered model can be reduced to a linear Sobolev-type equation

$$L\dot{u} = Mu + f. \tag{4}$$

Equation (4) will be considered with *multipoint initial-final condition*

$$\begin{gathered} P_j(u(\tau_j) - u_j) = 0, \quad u_j \in \mathfrak{U}, \quad j = \overline{0,n}, \\ \tau_j \in (a,b), \quad \tau_{j-1} < \tau_j, \quad j = \overline{1,n}. \end{gathered} \tag{5}$$

All reviews are conducted in Banach spaces $\mathfrak{U}$ и $\mathfrak{F}$ and the operators L, $M \in \mathcal{L}(\mathfrak{U};\mathfrak{F})$, a P_j these are relatively spectral projectors.

To carry out a computational experiment, the eigenvalues and eigenfunctions $\varphi_k(x)$ of the Sturm — Liouville problem are calculated on an eight-edge geometric graph. We will give the main stages of the algorithm using a specific example.

Step 1. Consider one intersection (see fig. 1). Entering the input data: $\lambda = 10; \nu = 1; f_k(x) = 0; l_1 = l_2 = \pi$, initial speeds

$$u_{00} = \left(5\sin(x + \frac{\pi}{2}) - 2, 5\sin(4x + \frac{\pi}{2}) - 2\right),$$

$$u_{01} = \left(10\sin(x - \frac{\pi}{2}) + 3, 10\sin(2x - \frac{\pi}{2}) + 3\right).$$

Step 2. We represent the solution in the form of a Galerkin sum. The resulting equation is transformed by multiplying by $\varphi_k(x)$ scalarly. A system of algebraic - differential equations is compiled. For each edge, multipoint initial-final conditions for the system of equations are compiled.

Step 3. A system of algebraic - differential equations with the corresponding initial data and conditions of a multipoint initial-final problem is solved and the found Galerkin coefficients are substituted into the approximate solution. The solution is displayed in the form of a graph (see fig. 2, 3).

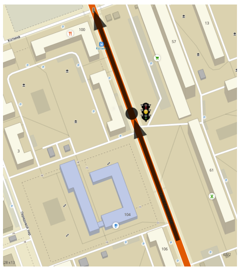

Figure 1. Road map

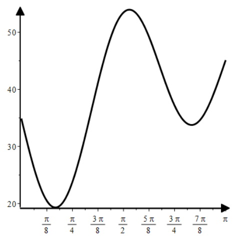

Figure 2. Schedule of solving problem (1)–(3), (5) before changing the traffic signal

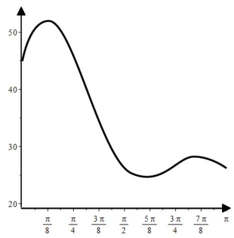

Figure 3. Schedule of solving problem (1)–(3), (5) after changing the traffic signal

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Свиридюк Г. А., ***Загребина С. А.***, ***Конкина А. С.*** (Челябинск, ЮУрГУ). **Модифицированная модель транспортного потока на перекрестке.**

Резюме: В настоящее время одной из важных проблем мегаполиса является управление дорожным движением, а в связи с проблемой образования предзаторных и заторных ситуаций в населенных пунктах, соответственно, эти исследования являются актуальными. В данной работе описывается вычислительный эксперимент для математической модели транспортного потока.

Ключевые слова: Многоточечная начально-конечная задача, геометрический граф, транспортные потоки, уравнение Осколкова.