ОБОЗРЕНИЕ
ПРИКЛАДНОЙ И ПРОМЫШЛЕННОЙ
Том 30 МАТЕМАТИКИ Выпуск 2
2023

**V. G. Vysotina** (Moscow, Review of applied and industrial mathematics). **The numerical modeling of a turbulence evolution in a swirling flow inside a long pipe.**

UDC 533 DOI https://doi.org/10.52513/08698325_2023_30_1_1

*Resume*: Numerical research of reorganization of a structure of the swirling laminar flow of air in structure of the swirling laminar flow, with the turbulence, consisting of two streams moving towards each other and divided by border, has been undertaken to try to understand the mechanism of the processes leading to such reorganization. Godunov's method was used for analysis of unsteady process of an establishment of decision in time. During process of an establishment of the decision, it has been opened, that the reason of formation of a structure of the double laminarno-turbulent swirling flow containing "vortex breakdow" and dividing by border, is a movement of the swirling flow towards higher pressure on an exit than pressure on an entrance is.

*Keywords*: air, vortex breakdown, swirling, structure, a constant angle of swirling, a long pipe, and Godunov's method, establishment of decision in time, double laminarno-turbulent swirling flow

The unsteady process in time of the swirling flow through an axisymmetric long direct pipe was investigated in this study. The steady state of decision for a swirling angles $\alpha = 17,62^\circ$ and $\alpha = 17,63^\circ$ at value of the relation of pressure $P_{out}/P_0 = 0,990$ was received. Godunov's used method and problem statement are published in [1]. Experimental modeling of the compressible swirling flow of air was carried out in axisymmetric pipe in length of 1 meter with external radius $R = 0,04m$ [2]. Airbraking parameters are $P_0 = 100500,\ 8$ Pas; $\rho_0 = 1,1945\,kg/m^3$; $\varkappa = 1,4$; $RG = 287,15m^2/(c^2\cdot{}^0K)$; $Re \approx 10^4 \div 10^6$. The type of swirling was $\mathrm{tg}(\alpha) = \mathrm{const}$ , i. e. the constant value of swirling angle was set up on height of entrance section of the channel. $C_n$ is a normal velocity at entrance section, is a swirling angle, and R is an external pipe's radius and $R_i$ is a flowing pipe's radius. The grid system contains 200 equally meshes on the length and 21 condensed meshes to an axis and external contour on the radius. The grid system consisting of $201 \times 21 = 4221$ points with 201 points in the $z$ direction and 21 points in the r direction.

Results of studying of vortex breakdown in a direct pipe for a constant angle at entrance section ( $\mathrm{tg}(\alpha) = \mathrm{const}$ ) by Godunov's method were compared to skilled data by Shigeo of Uchida [2] and Sarpkaya 1971a [3] and calculation by W. J. Grabowski and S. A. Berger [4].

The radial distributions of the axial and circumferential velocity components are presented to bubble vicinities in a long direct pipe (1 meter) at a swirling air flow receiving experimentally published in [2]. The calculated by Godunov's method profiles of axial and circumferential velocity components were compared to profiles of axial and circumferential velocity components before a bubble, in the middle and behind a bubble [2]. Comparison a photo of a axisymmetric bubble in the swirling flow of water in a pipe receiving by Sarpkaya [3] experimentally and a picture of stream function contours as a result of calculations of vortex breakdown in a pipe obtaining with using of stationary equations Navier-Stokes (Reynolds's number was equaled 200) by W. J Grabowski and S. A. Berger published in [4]. The axisymmetric bubbles calculated by Godunov's method were compared with [4, picture 2]. The results of [2, 3 and 4] visually qualitatively agreed with results of author's calculations [6].

Studying of the swirling flow of air at the swirling type $\mathrm{tg}(\alpha) = \mathrm{const}$ [5] has shown, that in a range of swirling angles $\alpha = 16^\circ \div 40^\circ$ there is a stream reorganization the expense inside an interval $\alpha = 16^\circ \div 40^\circ$ becomes negative value (picture 1 a) and the structure of swirling flow transformed. The structure for $\alpha = 17,62^\circ$ is a laminar swirling flow in the long direct pipe. The new structure for $\alpha = 17,63^\circ$ has two streams — one at the top wall, swirled, moving in a direction to exit section, and the second, moving in a direction to entrance section take place in the channel. As a result there are two different structures for swirling angle $\alpha = 17,62^\circ$ (picture 1 b) and for $\alpha = 17,63^\circ$ (picture 2).

The steady state decision of calculation is allowable if you got a result of iterations of solutions in time. Let a defined value error on pressure $Ep_z = 0,670 \cdot 10^{-6}$ for swirling angle $\alpha = 17,62^\circ$ and $Ep_z = 0,660 \cdot 10^{-4}$ for swirling angle $\alpha = 17,63^\circ$. And $Ep$ is flowing value error of pressure. If during the process of iterations the flowing error on pressure $Ep$ in the field of calculation does not exceed $Ep_z$, i. e. the field of pressure at the given time step does not differ from a pressure field on the previous time step more than on $Ep_z$, calculation is over; if the error $Ep$ becomes equal or less $Ep_z$. ($Ep \leqslant Ep_z$) and the steady state decision of calculation has been achieved. In a case of a swirling angle $\alpha = 17,62^\circ$ a defined value error on pressure $Ep_z = 0, 0.67 \cdot 10^{-6}$ has been reached at number of iteration time steps Nt = 445400. In a case of swirling angle $\alpha = 17,63^\circ$ a defined error on pressure $Ep_z = 0,660 \cdot 10^{-4}$ has been reached at number of iteration's time steps $Nt = 1137800$.

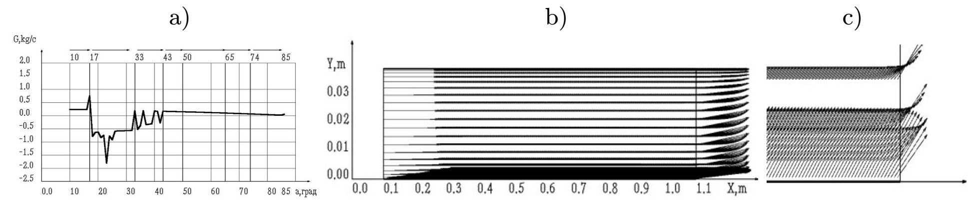



Fig. 1. Change of the expense of air $G(kg/c)$ depending on a swirling angles setting on an entrance section. Field of velocity's vectors for a swirling angle $\alpha = 17,62^\circ$. c) The fragment of a field of velocity's vectors on about an axis at exit section.

In a fig. 1 a change of the expense of air $G(kg/c)$ depending on a swirling angles in a range of swirling angles $\alpha = 10 \div 87^\circ$. is shown. Swirling angles $\alpha$ were set on at an entrance section. The steady state decision is a laminar swirling flow in the long direct pipe has unique feature is the thickening of lines of a current about the pipe's axis. The field of velocity's vectors in the channel and fragment of velocity's vectors on about an exit section for a swirling angle $\alpha = 17,62^\circ$ are shown in figures 1б and 1c.

Reorganization of a stream from a laminar swirling flow at $\alpha = 17,62^\circ$ to the divided laminar swirling flow at $\alpha = 17,63^\circ$. has occurred suddenly, as though jump, after change of value of a swirling angle on $0,01^\circ$. In the fig. 2 the solution having two streams in the channel is shown. The received decision contains border between the streams, representing classical contact rupture [7] at which gas through border does not flow, the pressure profile is continuous, but has an extremum point (the derivative is equal to zero), and profiles of density and velocity have break of continuity in a place of classical contact rupture (break). Process of receiving steady state solution for swirling angle $\alpha = 17,63^\circ$ has demanded $Nt = 1137800$ iteration's time steps. The field of vectors of velocities of such flow in the channel for закрутки $\alpha = 17,63^\circ$ is resulted in fig. 2. Profiles of pressure, density and axial velocity are shown in figure 3.

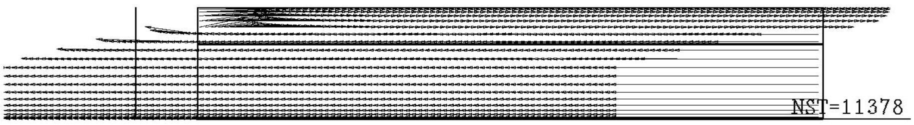



Fig. 2. A field of velocity's vectors with two streams divided by border, — one at the external radius, swirled on an entrance, moving in a direction to exit section, and the second at the pipe's axis, laminar, moving in a direction to entrance section. A swirling $\alpha = 17,63^\circ$.

a) b) c)

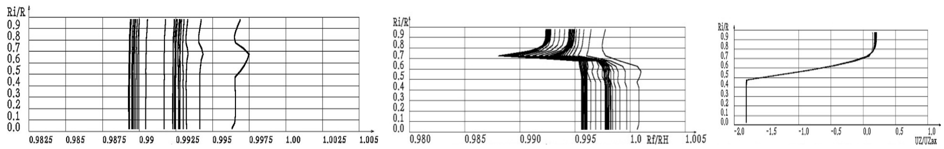


Fig. 3. Tree profiles: a) the profile of pressure having a point of an extremum; b) the profile of density having a point of catastrophe of continuity; c) the profile of axial velocity having a point of a cusp. A swirling angle $\alpha = 17,63^\circ$.

The establishment of the solution (getting a steady state solution) has taken place at a step on time $Nt = 445400$ for a swirling flow $\alpha = 17,62^\circ$. And for a swirling angle $\alpha = 17,63^\circ$ the steady state solution has occurred at a step on time $Nt = 1137800$. The process of decision's establishment according to change of an error on pressure $Ep$ for $\alpha = 17,62^\circ$ (a) and for $\alpha = 17,63^\circ$ (b) is shown in fig. 4.

a) b)

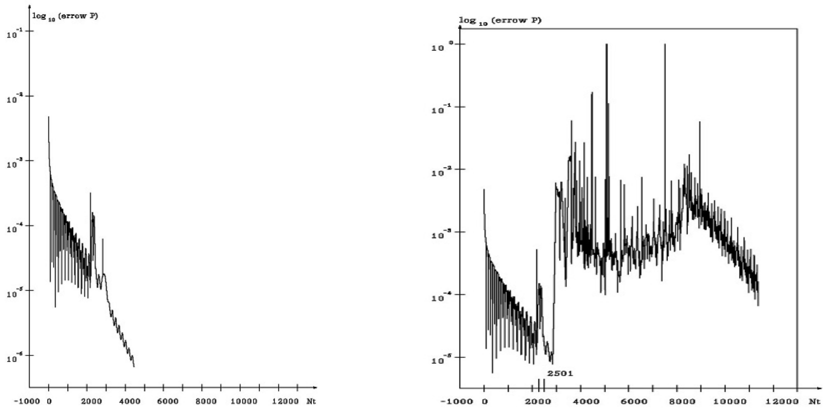



Fig. 4. The iteration's time steps of an establishment of the decision on pressure's error $Nt = 445400$ for a swirling angle $\alpha = 17,62^\circ$ (a) and $Nt = 1137800$ for a swirling angle $\alpha = 17,63^\circ$ (b).

To follow the track of solution's establishment in time from a steady state of a air in a rest, when there is no motion inside a pipe, to the a result of calculations, a steady state of a swirling flow of air, which in fig. 3 and 4 are shown, it seems, as the same process as photographing or filming during in any time, receiving intermediate snapshots.

On fig. 5 a curves of average values of pressure on an entrance section and at an exit section from the channel depending on an intermediate time steps Nt, are shown. The curve of a gradient of pressure (Pavexit – Pavinput) is represented in fig. 5 b.

a) b)

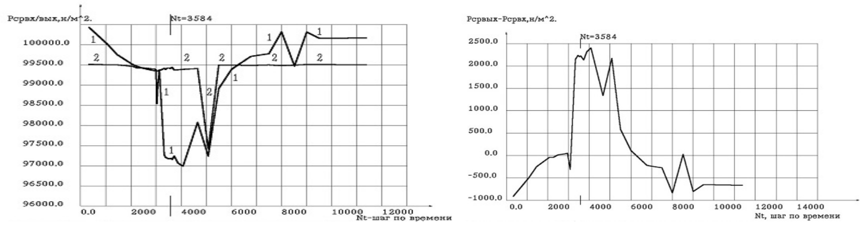



Fig. 5. Curves pressure $P_{avinput}$ — 1 and $P_{avexit}$ — 2 is in fig. 5). A curve of gradient of pressure ( $P_{avexit}-$ P $_{avinput}$ ) in the process of a decision establishment in time is in fig. 5 b. The time steps changed from $Nt = 34100$ to $Nt = 1137800$. A swirling angle $\alpha = 17,63^\circ$.

From the analysis of graphs follows, that in ranges of intermediate steps $Nt = 34100 \div 228100$ and $Nt = 800600 \div 1137800$ average values of pressure on an entrance section (fig. 5 a, curve 1) above values of average values of pressure on an exit section (fig. 5 curve 2). These ranges are areas, where a pressure gradient $(P_{avexit} - P_{avinput}) \leqslant 0,00$ (fig. 5 b). According to signs (symptoms) of turbulence of L. Prandtl [8], in these ranges of time steps the flow should be laminar. In a range of time steps $Nt = 300000 \div 600000$ average values of pressure on an exit section (fig. 5 a, curve 2) above than average values of pressure upon an entrance section (fig. 5 a, curve 1), i. e., agree to L. Prandtl, if the air swirling flow moving towards the more height pressure, the turbulent flow could be takes place. In this range of time steps $Nt = 300000 \div 600000$ a pressure gradient $(P_{avexit} - P_{avinput}) > 0,00$ (fig. 5 b) and an air laminar swirling flow becomes an air laminar swirling flow with turbulent sites.

The graph of change of an average value tangential component angulation velocity, tangential component of vortex, is $W_\theta = \partial C_r/\partial z - \partial C_z/\partial r$, depending on a time steps of an establishment of the decision, for the swirling angle $\alpha = 17,63°$, which set on an entrance section, are represented on fig. 6 a.

Reynolds's number as the relation of an inertia forces to a friction forces, calculated as $Re = U \cdot R/v$, where $U(m/s)$ is an average value velocity on an entrance section, $R(m)$ is a radius, $v = \mu/\rho$ is factor of kinematic viscosity, $\mu = 1,780 \cdot 10^{-5}$ $(n \cdot sec/m^2)$ is factor of dynamic viscosity, $\rho\,(kg/m^3)$ is density. The inertia forces calculated as the sum of tangential and centrifugal forces $U^2/R(kg \cdot m/s^2)$. The centrifugal force is $\rho\, U_{T^2}/R$, $U_T$ -average value centrifugal velocity on an entrance section. The tangential force is residual of subtraction the centrifugal force from inertia forces. The friction forces calculated using the formula $\mu U/R^2$.

The graphs of curve of an average values of tangential component of vortex $W_\theta$ (fig. 6 a), the curves of an average values of numbers of Reynolds (fig. 6 b), the curves of an average values of tangential and centrifugal forces (fig. 6 c) in entrance and exit sections of the channel during a process of a decision establishment in time are presented in a fig. 6.

a) b) c)

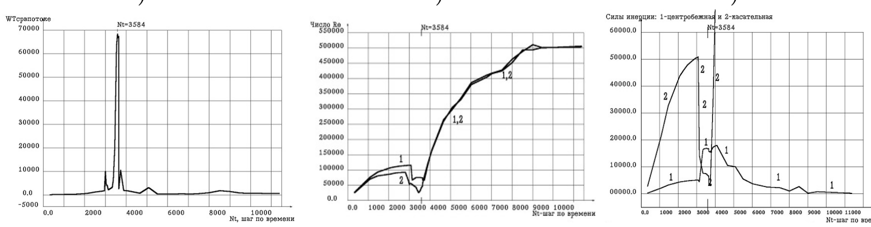


Fig. 6. A curve of an average values of tangential component of vortex Wavinflow (a); the curves of average values of number of Reynolds on entrance section Reavinput (1) and exit section Reavexit (2) of the channel (b); the curves of centrifugal (1) and a tangential (2) forces during in the process of a decision establishment in time. The time step changed from $Nt = 34100$ to $Nt = 1137800$. A swirling angle is $\alpha = 17,63°$.

In a range of time steps $Nt \approx 304000 \div 408000$, the curve of tangential component of vortex $W_\theta$ (fig. 6 a) has a maximum, and both curves $Re_{avinput}$ and $Re_{avexit}$ (fig. 6 б) have a minimum. In the same range of time steps there is a minimum a curve of average value pressure on entrance section Pevinput (fig. 5a) and a maximum of a pressure's gradient $(P_{evexit} - P_{evinput})$ (fig. 5 б). The curves of tangential and centrifugal forces are represented in fig. 6 c. In a range of time steps $Nt \approx 341000 \div 308000$, average values of both forces increase, a curve of a tangential force much more, than centrifugal. In a range of time steps $Nt \approx 308000 \div 358400$ there is a sharp reduction of values of a tangential force and increase in values of the centrifugal force. The sharp increase in values of a tangential force in a range of steps $Nt \approx 358400 \div 408000$ and reduction of values of centrifugal force testifies that a tangential force as a part of inertial forces became prevailing whereas centrifugal force decreases practically to zero values by the end of process of an establishment of the decision on time (see fig. 6 c).

Let's consider consistently a change of fields of velocities and other parameters during the time of the process of an establishment of the solution.

In fig. 7 a the field of velocity's vectors of a laminar swirling flow for a time step $Nt = 208100$ is shown. A fragment of this field of velocity (fig. 6 б) at a channel axis where there is a condensation of lines of a flow and increase in values of velocity is shown.

a) b)

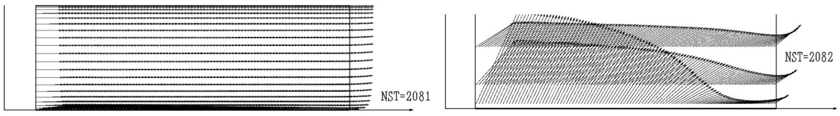



Fig. 7. A field of velocity's vectors for a time step $Nt = 208100$ (a) and a fragment of a field at a channel axis (b). A swirling angle $\alpha = 17,63°$.

In a range of steps $228100 \div 304100$ average value pressure on an exit section $P_{avexit}$ and an entrance section $P_{avinput}$ become equal (fig. 5), and a pressure gradient $(P_{avexit} - P_{avinput}) \approx 0.0$. In fig. 8 a the field of velocity's vectors, a laminar swirling flow for time step $Nt = 250100$ is shown. In fig. 8 б the fragment of a field of velocity's vectors is shown, where a small zone of returned swirling flow at the axis beneath an exit section is formed.

a) b)

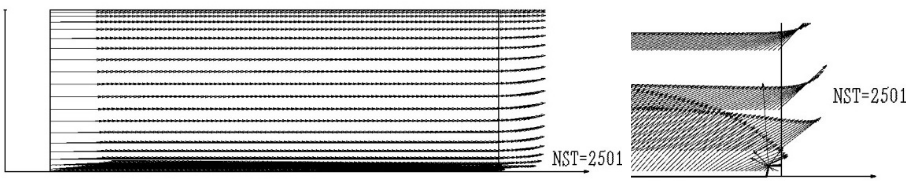



Fig. 8. A field of velocity's vectors (a) and a fragment of a field of velocity's vectors at a channel axis near an exit section (b). A time step $Nt = 250100$. A swirling angle $\alpha = 17,63°$.

The further formation of zones of returned flow with construction separation's borders in the laminar swirling current at the axis of a pipe is shown in fig. 9. The fragments of fields of velocity's vectors for time steps $Nt = 228100$ (a), 295100 (b), 300000(c), 308100 (d) at a channel axis near an exit section are presented. During increase of time steps the border is moving towards entrance section from an exit section. The field of velocity's vectors of swirling flow is not laminar current, it is turbulent flow.

a) b) c) d)

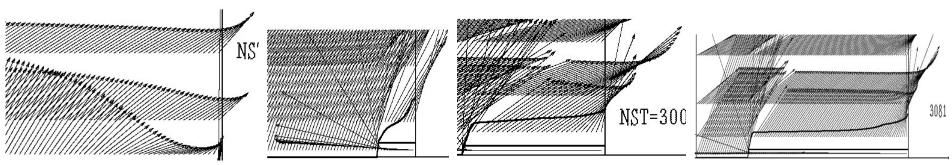



Fig. 9. Fragments of a field of velocity's vectors with border of separation at an axis of the channel for time steps $Nt = 228100$ (a), 2950100 (b), 300000 (c), 308100 (d). A swirling angle $\alpha = 17,63°$.

A field of velocity's vectors of laminar swirling flow, dividing on two streams by border between them, is shown in fig. 10 a. There is a fist full stream from an exit of the channel to an entrance. The construction the border between the basic laminar swirling stream and the first laminar stream at an axis, which flowing from exit to entrance is finished, it is well visible on fig. 10 b.

a) b)

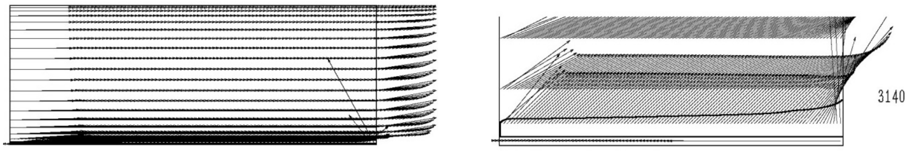



Fig. 10. A field of velocity's vectors (a) and a fragment of a field of velocity's vectors at a channel axis (b). Border formation between two streams is finished. A time step is 314000. An angle of swirling is $\alpha = 17,63°$.

In a range of time steps $Nt = 333200 \div 550300$, there is a swirling flow moving towards the more higher values of pressure on an exit section $P_{avexit}$ than values of pressure on an entrance section $P_{avinput}$, and a pressure gradient is $(P_{avexit} - P_{avinput}) < 0$ (fig. 5). In a range of time steps $Nt = 333200 \div 550300$ a curve of tangential component of vortex W? has a maximum $(Nt = 358400)$, a curve of Reynolds's numbers have a minimum $(Nt \approx 314000 \div 388100)$, and values of centrifugal force exceed values of tangential force $(Nt \approx 333200 \div 373200)$. The process of a further formation of a returned zone and border of separation is continuing. The zone's border moves towards an entrance section and upwards on external radius of a pipe. The field of velocity's vectors and fragment of a field for $Nt = 388100$ are presented in fig. 11. The laminar current has areas (sites) of turbulence about border of a zone of a separation.

a)

b)

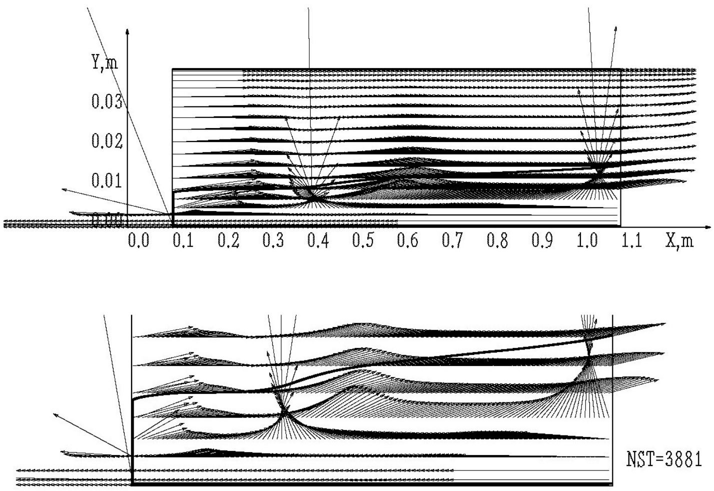



Fig. 11. A field of velocity's vectors (a) and a fragment of a field at a channel axis (b). A laminar swirling flow with turbulence areas (sites) is shown. A time step is $Nt = 363800$. A swirling angle A field of velocity's vectors (a) and a fragment of a field at a channel axis (b). A laminar swirling flow with turbulence areas (sites) is shown. A time step is $Nt = 363800$. A swirling angle $\alpha = 17,63°$.

Pressure gradients in entrance section $DP_{entrance}$ (1) and at axis $DP_{axis}$ (2) of the channel are shown in the fig. 12. In a range of time steps $Nt = 304100 \div 408200$ pressure gradients DPentrance and DPaxis are positive, considerably increase and reach the maximum value at time step $Nt = 408100$. The gradient of pressure $DP_{axis}$ has the considerable negative values promoting occurrence of a separation of a stream from an axis in a range time steps $Nt = 304100 \div 314000$. It is really, the separation zone appears (fig. 8, $Nt = 250100$ ) and develops (fig. 11, $Nt = 250100 \div 363800$ ). Consequently, there are the returned adverse pressure gradients $DP_{entrance}$ and $DP_{axis}$ leading to occurrence of a returned flow and a separation zone in a range of steps $Nt = 304100 \div 408200$. In range of time steps $Nt = 408200 \div 900700$ there are the strong pressure favorable a gradients $DP_{entrance}$ and $DP_{axis}$ which gradually decreases (fig. 12). Finally, the steady state of decision for a swirling angles A field of velocity's vectors (a) and a fragment of a field at a channel axis (b). A laminar swirling flow with turbulence areas (sites) is shown. A time step is $Nt = 363800$. A swirling angle $\alpha = 17,63^\circ$ is established, a returned stream at pipe's axis and its border are formed, the new structure having two streams in the channel is received. Process of formation of a returned flow at pipe's axis and its border is stretched and finally the air swirling flow is established in range $Nt = 900700 \div 1137800$ and becomes practically laminar (fig. 2). Ultimately, three areas of structures were explored. Two areas in a range of time steps $Nt = 34100 \div 228100$ and $Nt = 800600 \div 1137800$ where are a steady laminar swirling flow with insignificant sites of turbulence. The third area in a range of time steps $Nt = 228100 \div 800600$ where there is an unsteady laminar swirling flow with significant sites of turbulence around a constructing border.

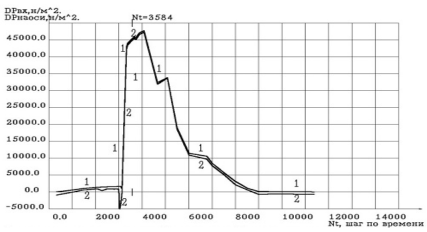
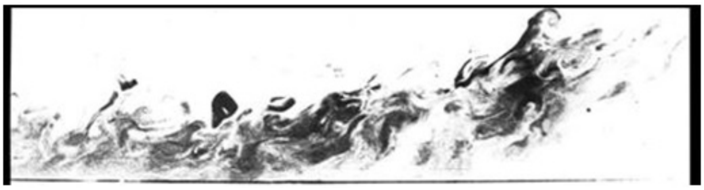



Fig. 12. Pressure gradients in entrance section $DP_{entrance}$ — 1 and at axis $DP_{axis}$ — 2 in the channel during a process of a solution's establishment in time are shown. Time steps $Nt = 34100 \div 1137800$. A swirling angle $\alpha = 17,63^\circ$.

On fig. 13a and 14 a are two photos R. E. Falco from an album M. van Dyke [9, fig. 164]. On a photo fig. 13a are a separation of a turbulent interface which is formed at a strong returned gradient. In fig. 13 б is the fields of velocity's vectors and a separation zone, receiving in the process of a decision establishment for time step $Nt = 408100$.

In fig. 14 a is the second photo where it a becoming laminar interface. The field of v elocity's vectors of a laminar swirling flow with the turbulence sites, having border between the streams, formed at gradually decreasing strong favorable gradient of pressure, is shown on time step $Nt = 950700$, fig. 14 b.

a)



b)

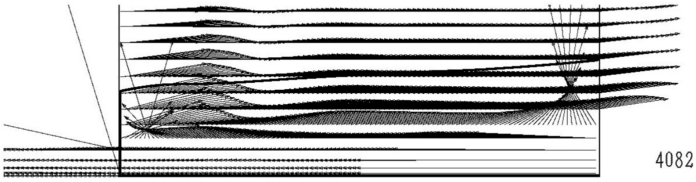



Fig. 13. A field of velocity's vectors (a) and a fragment of a field at a channel axis (b). A time step is $Nt = 408100$. There is a laminar swirling flow with turbulence sites, having border formation between two streams. There is a visual comparison from photo R. E. Falco from album M. van Dyke [9]. A swirling angle is $\alpha = 17,63°$.

a)

b)

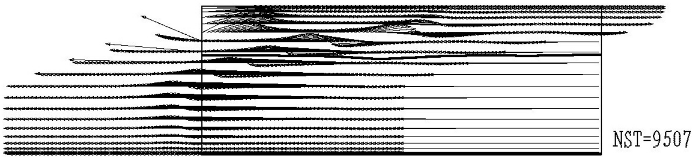



Fig. 14. Photo R.E. Falco from album M. van Dyke [9]. A field of velocity's vectors of a laminar swirling flow with turbulence sites, having border formation between two streams. A time step is $Nt = 950700$. A swirling angle is $\alpha = 17,63°$.

## CONCLUSIONS

Numerical research of unsteady process of an establishment of the decision in time for an air swirling flow in a long pipe brought out that:

1. Establishment process, which was done for a swirling angle $\alpha = 17,63°$, has three areas of ranges of time steps (fig. 5). There are two areas where average pressure on an entrance section more height (above) or equal to an average pressure on an exit section. It is an area of time steps $Nt = 34100 \div 308100$, where there is a laminar swirling flow and an area of time steps $Nt = 900700 \div 1137800$ where there is a laminar swirling flow, having elements of turbulence, divided in to two opposite moving streams.

2. The third area of time steps $Nt = 308100 \div 900700$, in which average pressure on an exit section more height than average pressure on an entrance section (fig. 5). The air swirling flow is moving towards the more height pressure on an exit

section. Occurrence of a returned stream at an pipe's axis and the beginning of border formation between opposite moving streams take place owing to occurrence and growth of the big returned adverse gradients of pressure in entrance section of the channel and on channel axis at $Nt = 308100 \div 408200$. The strong favorable gradient of pressure gradually decreasing at $Nt = 408200 \div 900700$ stretching process of formation of a returned stream and its border, since the stream continues to move towards the more height pressure on an exit section (fig. 12).

3. In a range of time steps $Nt = 900700 \div 1137800$ where pressure on an entrance section above than pressure on an exit section, the flow established and became definitive practically laminar. The finally decision is a structure of an air swirling flow consisting of two opposite streams with insignificant elements of turbulence. Time steps of solution's establishment are $Nt = 1137800$ (fig. 2).

4. The numerical modeling of unsteady process (process of establishment) in time, process of transformation one structure of steady state of flow to another structure of steady state of flow, which differs from the first structure, was made.

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